

1 CHAPTER OVERVIEW

Your visual roadmap through the chapter

Annuities & Perpetuities 1

Regular equal payments — ordinary, due, and forever

Net Present Value 2

PV of inflows — PV of outflows; accept if positive

Internal Rate of Return 3

The discount rate at which $NPV = 0$

Payback Period 4

Time to recover the initial investment

WHAT IT'S ABOUT

Discounted cash flow (DCF) evaluates whether proposed investments are financially worthwhile, using annuities, perpetuities, NPV, IRR and payback period techniques.

WHY IT MATTERS FOR QAB

DCF techniques are the standard tools for capital-investment decisions — comparing project returns against the cost of capital and each other.

EXAM SKILLS REQUIRED

Apply annuity/perpetuity formulas; compute NPV and accept/reject a project; estimate IRR by interpolation; calculate payback and discounted payback period.

2 KEY DEFINITIONS

Source-based terminology, exam-ready

Annuity

A series of regular periodic payments of equal amount — a constant cash flow for a given number of time periods.

Key word: equal, regular, finite

Ordinary Annuity / Annuity Due

Ordinary: payments in arrears (end of each period). Due: payments in advance (beginning of each period).

Key word: end vs start

Perpetuity

A constant annual cash flow "forever" — an annuity in which the cash flows continue into the long-term future indefinitely.

Key word: forever

Sinking Fund

Setting aside a fixed sum at regular intervals to achieve a specific target sum at a future point in time.

Key word: save toward a target

Net Present Value (NPV)

The difference between the present value of cash inflows and the present value of cash outflows over a project's life.

Key word: PV in — PV out

Internal Rate of Return (IRR)

The discount rate that makes a project's NPV exactly equal to zero.

Key word: $NPV = 0$ rate

Payback Period

The length of time it takes for an investment's cash inflows to recover its initial cost.

Key word: time to break even

Discounted Payback Period

Like payback period, but the future cash flows are discounted to present value first, accounting for the time value of money.

Key word: payback, discounted

3 CORE CONCEPT

Explained in exam-oriented language

CONCEPT → Annuities

MEANING

A stream of equal payments; can be valued as a future sum (FV) or a present value (PV).

PURPOSE

Avoid discounting/compounding each payment individually — use one annuity factor instead.

APPLICATION

Loan installments, savings plans, sinking funds, project cash flows.

CONCEPT → NPV / IRR

MEANING

NPV measures value created at a given cost of capital; IRR finds the rate at which value created is zero.

PURPOSE

Decide whether a project is worth undertaking.

APPLICATION

Capital budgeting / investment appraisal decisions.

4 KEY RULES & PRINCIPLES

Every source rule, exam-ready

RULE 1 Recognising annuity due

- If payments start now/immediately, or the first payment is at the beginning of the first period, it is an **annuity due**

RULE 4 NPV decision rule

- NPV > 0 → accept (value created)
- NPV = 0 → break-even
- NPV < 0 → reject (value destroyed)

RULE 2 Annuity due vs ordinary relationship

- Sum (FV) of annuity due = $(1+i) \times$ sum of ordinary annuity
- PV of annuity due = $(1+i) \times$ PV of ordinary annuity
- Annuity due is always the larger figure, all else equal

RULE 5 IRR decision rule

- IRR > required rate of return → accept
- IRR = required rate → break-even (NPV = 0)
- IRR < required rate → reject

RULE 3 Perpetuities

- Sum (FV) of a perpetuity cannot be determined (it is infinite)
- PV of an ordinary perpetuity = $R \div i$
- PV of a perpetuity due = $R + (R \div i)$

RULE 6 Payback period

- A shorter payback period is generally preferred
- Ignores the time value of money, and any cash flows after payback — use discounted payback to correct for time value

5 FORMULA ZONE — ANNUITIES & PERPETUITIES

Every formula supported by the source text

SUM (FV) OF ORDINARY ANNUITY

$$S_o = R [(1+i)^n - 1] \div i$$

R=periodic payment, i=rate per period, n=number of payments; payments at end of each period.

SUM (FV) OF ANNUITY DUE

$$S_D = (1+i) \times S_o$$

Payments at the start of each period; always > sum of ordinary annuity.

PV OF ORDINARY ANNUITY

$$P_o = R [1 - (1+i)^{-n}] \div i$$

Used for loan repayments and valuing a stream of receipts/payments in arrears.

PV OF ANNUITY DUE

$$P_D = (1+i) \times P_o$$

Always > PV of ordinary annuity, all else equal.

PV OF PERPETUITY (ORDINARY)

$$P_o = R \div i$$

Sum (FV) of a perpetuity cannot be determined.

PV OF PERPETUITY DUE

$$P_D = R + (R \div i)$$

The first payment (R) is received immediately, plus the PV of the remaining ordinary perpetuity.

5 FORMULA ZONE — NPV, IRR & PAYBACK

Every formula supported by the source text

NET PRESENT VALUE

$$\text{NPV} = \text{PV of cash inflows} - \text{PV of cash outflows}$$

Discount each period's cash flow at the cost of capital, then sum. Initial investment is a negative (outflow) at year 0.

IRR BY INTERPOLATION

$$\text{IRR} = A\% + \left[\frac{\text{NPV}_A}{\text{NPV}_A - \text{NPV}_B} \right] \times (B - A)\%$$

NPV_A at rate A% should ideally be positive, NPV_B at rate B% negative, for best accuracy.

PAYBACK PERIOD (EVEN CASH FLOWS)

$$\text{Payback} = \text{Cost of Investment} \div \text{Average Annual Cash Flow}$$

For uneven cash flows, accumulate cash flows year by year until the investment is recovered.

DISCOUNTED PAYBACK PERIOD

$$\text{PV of Cash Flow} = \text{Cash Flow} \div (1+i)^n$$

Accumulate discounted cash flows year by year until they cover the initial investment.

6 CALCULATION ROADMAP

General path for any discounted cash flow question

1- Read Question

List every cash flow and its timing

2- Identify Type

Annuity, perpetuity, NPV, IRR, or payback?

3- Select Formula

Ordinary or due? Even or uneven cash flows?

4- Substitute

Put known values into the formula

5- Calculate & Interpret

Solve, apply decision rule, state answer

7 STEP-BY-STEP METHODS

Kept separate — do not mix methods

METHOD A · Sum (FV) or PV of an Annuity

- 1 Identify R, i (per period) and n
- 2 Decide ordinary (end of period) or due (start of period)
- 3 Substitute into the matching Sn or P formula
- 4 If "due", multiply the ordinary result by (1+i)

METHOD D · Net Present Value (NPV)

- 1 List every cash flow by year, including the initial investment (negative)
- 2 Discount each year's cash flow at the cost of capital
- 3 Sum all discounted cash flows to get the NPV
- 4 Apply the decision rule: accept if NPV > 0

METHOD B · Sinking Fund (Solve for R)

- 1 Identify the target future sum (Sn), i and n
- 2 Substitute into $S_n = R[(1+i)^n - 1] \div i$
- 3 Rearrange to solve for R (the periodic saving)
- 4 Alternative: find PV of the target, then divide by the annuity factor

METHOD E · Internal Rate of Return (IRR)

- 1 Calculate NPV at a first discount rate, A%
- 2 Calculate NPV at a second rate, B%, ideally giving the opposite sign
- 3 Substitute both NPVs into the interpolation formula
- 4 Compare the estimated IRR to the required rate of return

METHOD C · PV of a Perpetuity

- 1 Identify R (periodic payment) and i (period rate)
- 2 If payments are less frequent than annual (e.g. every 2 years), find the equivalent period's effective rate first
- 3 Substitute into $P = R \div i$ (ordinary) or $P = R \div (R+i)$ (due)
- 4 State the present value

METHOD F · Payback / Discounted Payback

- 1 List cash flows by year (discount them first if using discounted payback)
- 2 Build the cumulative cash flow, year by year
- 3 Find the year the cumulative total turns positive
- 4 Add the fraction of that year needed: remaining amount ÷ that year's cash flow

8 DIFFERENT CASES / SCENARIOS

Recognise the wording, pick the right route

IF payments occur at the END of each period

→ USE **ordinary annuity** formulas

IF payments start NOW / immediately / at the beginning

→ USE **annuity due**: multiply ordinary result by $(1+i)$

IF the cash flow continues forever

→ USE **perpetuity**: $P=R+i$ (only PV can be found)

IF a target future sum must be reached by saving

→ USE a **sinking fund**: solve for R

IF NPV > 0

→ **Accept** the project

IF NPV < 0

→ **Reject** the project

IF cash flows are equal every year

→ Payback = Investment ÷ Annual Cash Flow

IF cash flows are uneven, or time value of money matters

→ USE the **cumulative (discounted) cash flow** method

9 COMPARISON TABLES

Don't confuse these!

	Ordinary Annuity	Annuity Due
Payment timing	End of each period (in arrears)	Start of each period (in advance)
Sum (FV)	$So = R[(1+i)^n - 1] \div i$	$SD = (1+i) \times So$
Present Value	$Po = R[1 - (1+i)^{-n}] \div i$	$PD = (1+i) \times Po$

	Net Present Value (NPV)	Internal Rate of Return (IRR)
What it finds	The rupee value created at a given cost of capital	The discount rate at which NPV = 0
Decision rule	Accept if NPV > 0	Accept if IRR > required rate of return
Method	Discount all cash flows at one given rate	Interpolate between a positive and negative NPV

	Payback Period	Discounted Payback Period
Time value of money	Ignored	Accounted for (cash flows discounted first)
Cash flows used	Raw (undiscounted) cash flows	$PV \text{ of cash flow} = \text{Cash Flow} \div (1+i)^n$
Result	Shorter, since money isn't discounted	Longer, since future cash flows are worth less today

10 READING CASH-FLOW TIMELINES & TABLES

How to interpret the source's diagrams

Cumulative cash flow timeline (payback)

Plot the running (cumulative) total below each year. The payback point is where the cumulative total crosses from negative to positive. Interpolate within that year: $\text{fraction} = \frac{\text{remaining negative balance}}{\text{that year's cash flow}}$.

NPV discount-factor table

Each row is a year's cash flow, multiplied by that year's discount factor (from the rate column) to give its present value. Sum the present-value column (inflows and the negative initial outflow) to get the NPV.

Annuity factor tables

An annuity factor is the sum of the individual discount factors for a run of periods. Read it at the intersection of the correct rate column and period row, then multiply by the periodic payment (or divide the target/loan by it to find the payment).

IRR interpolation diagram

NPV is plotted against discount rate; the true relationship curves, but interpolation assumes a straight line between the two calculated NPVs to estimate where the line crosses zero (the IRR).

11 WORKED CALCULATION LOGIC

Method students must reproduce in the exam

GIVEN → FORMULA → SUBSTITUTION → CALCULATION → ANSWER

A company with a 10% cost of capital considers a project: initial payment Rs. 10,000; yields Rs. 6,000/year for 2 years. Should it be undertaken?

GIVEN: Year 0 = (10,000); Year 1 & 2 = 6,000 each; $r = 10\%$

FORMULA: $NPV = \sum (\text{Cash Flow} \times \text{Discount Factor})$

SUBSTITUTION: Year 1: $6,000 \times 1/(1.1)^1 = 5,456$; Year 2: $6,000 \times 1/(1.1)^2 = 4,959$

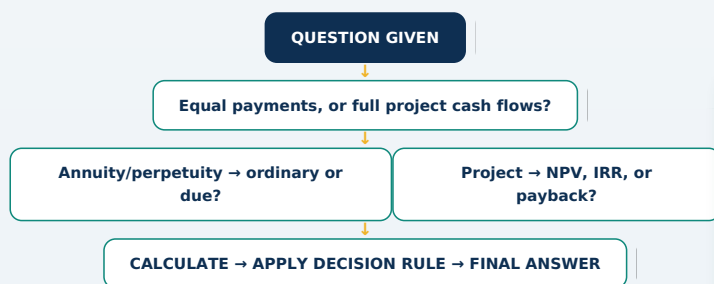
CALCULATION: $NPV = -10,000 + 5,456 + 4,959 = 415$

FINAL ANSWER: NPV = Rs. 415 (positive) — the project is viable and should be undertaken

12 EXAM QUESTION PATTERNS

- **Annuity/perpetuity calculation** — FV, PV, ordinary vs due
- **Sinking fund** — solve for the periodic saving amount
- **NPV calculation** — compute and apply accept/reject rule
- **IRR estimation** — interpolate between two discount rates
- **Payback / discounted payback** — even and uneven cash flows
- Reading annuity/discount factor tables

14 QUICK DECISION TREE



13 EXAM TRAPS

⚠ WATCH OUT

- ✗ **Ordinary vs due mix-up** — forgetting to multiply by $(1+i)$ for an annuity/perpetuity due.
- ✗ **Perpetuity sum** — trying to calculate a future sum for a perpetuity; only PV can be found.
- ✗ **IRR sign errors** — mixing up NPV_A and NPV_B or their signs in the interpolation formula.
- ✗ **Payback ignoring time value of money** when the question specifically asks for discounted payback.
- ✗ **Wrong cash flow sign** — treating the initial investment as positive instead of a negative outflow.

CHAPTER 6 IN 60 SECONDS

Ordinary Sn: $R[(1+i)^n - 1]/i$
Due Sn/Pn: $(1+i) \times \text{ordinary}$
Ordinary Pn: $R[1 - (1+i)^{-n}]/i$
Perpetuity PV: R/i
NPV > 0 accept, < 0 reject
IRR: $A\% + [NPV_a / (NPV_a - NPV_b)] \times (B - A)\%$
Payback = Investment ÷ Annual CF
Trap: "starts now" = annuity due