

1 CHAPTER OVERVIEW

Your visual roadmap through the chapter

Linear Programming

Inequalities, terminology, graphical method, corner-point & slope methods

1

LP of Business Problems

Formulating constraints & objective functions from real scenarios

2

Marginal Analysis

Marginal cost, marginal revenue, marginal profit

3

WHAT IT'S ABOUT

An optimization technique that finds the best (maximum or minimum) value of a linear objective function, subject to a set of linear constraints (inequalities).

WHY IT MATTERS FOR QAB

Product-mix, resource-allocation and cost-minimisation decisions are all classic business problems solved using linear programming's graphical approach.

EXAM SKILLS REQUIRED

Translate word problems into inequalities; plot constraints; identify the feasible region; find the optimum using corner points or slopes; spot redundancy/unboundedness.

2 KEY DEFINITIONS

Source-based terminology, exam-ready

Linear Programming Problem

A problem requiring the optimum (minimum or maximum) value of a function, subject to certain restrictions.

Key word: optimize + restrictions

Objective Function

In an LP problem, the function to be maximized or minimized.

Key word: maximize/minimize

Constraints

The set of restrictions, given by linear inequalities, that the solution must satisfy.

Key word: restrictions

Feasible Region

The area containing all the (usually infinite) combinations of x and y that satisfy every constraint — the feasible solutions.

Key word: all valid combinations

Optimum Solution

The one feasible point, out of infinite feasible points, that gives the maximum or minimum value of the objective function.

Key word: the best point

Positive (Non-Negativity) Constraints

$x \geq 0$ and $y \geq 0$. These limit the feasible region to the first quadrant and do not need to be plotted — they are the axes themselves.

Key word: $x, y \geq 0$

Corner Points

The points where the borders (edges) of the feasible region cross one another.

Key word: where edges meet

Redundant Constraint

A constraint whose removal does not change the feasible region, and so cannot affect the optimal solution.

Key word: removable, no effect

3 CORE CONCEPT

Explained in exam-oriented language

CONCEPT → Linear Programming

MEANING

An "optimization technique" — taking linear inequalities relating to a situation and finding the "best" value obtainable.

APPLICATION

Maximum profit from a product mix limited by resources; minimum cost across alternative inputs.

PURPOSE

Find the best combination of x and y that maximizes or minimizes an objective function.

CONCEPT → Feasible Region & Optimum

MEANING

The shaded area representing all combinations of x, y allowed by the constraints.

PURPOSE

The optimum always lies at a corner point of the feasible region's outer edge.

APPLICATION

Test each corner in the objective function (corner point theorem), or compare slopes.

4 KEY RULES & PRINCIPLES

Every source rule, exam-ready

RULE 1 The 6-step LP approach

- 1. Define variables
- 2. Construct inequalities for constraints
- 3. Plot the constraints on a graph
- 4. Identify the feasible region
- 5. Construct the objective function
- 6. Identify the optimum x, y

RULE 4 Wording → inequality sign

- Maximum limit / "cannot exceed" / "no more than" → use $\leq$
- Minimum limit / "at least" / "must supply no less than" → use $\geq$
- A range (e.g. "5 to 7 units") → write as a double inequality (both $\geq$ and $\leq$)

RULE 2 Plotting an inequality line

- If it has x- and y-intercepts: ignore the inequality sign, treat as an equation, set x=0 to find the y-intercept and y=0 to find the x-intercept, then join the two points
- If it passes through the origin (no intercepts): pick any two points that satisfy the equation and join them

RULE 5 Maximizing vs minimizing

- Maximizing (e.g. profit): optimum is the corner point furthest from the origin
- Minimizing (e.g. cost): optimum is the corner point closest to the origin

RULE 3 Shading the correct side

- Test the origin (0,0) in the inequality
- If it satisfies the inequality → shade the side near the origin
- If it does not satisfy the inequality → shade the side away from the origin

RULE 6 Redundancy & unboundedness

- A constraint is redundant if removing it leaves the feasible region unchanged
- An LP is unbounded if the objective function can improve without limit inside the feasible region — no finite optimal solution exists
- If no combination satisfies every constraint together, there is no feasible region / no feasible solution

5 FORMULA ZONE

Every formula supported by the source text

OBJECTIVE FUNCTION (GENERAL FORM)

$$Z \text{ (or } C) = c_1x + c_2y$$

c_1, c_2 = contribution/profit or cost per unit of x and y. Maximize for profit/contribution; minimize for cost.

CORNER POINT THEOREM

Substitute each corner (x,y) into Z → compare results

The optimum solution is the corner giving the highest Z (maximize) or lowest Z (minimize).

SLOPE COMPARISON METHOD

Rearrange each line into $y = (\text{intercept}) + (\text{slope})x$

The optimum lies at the intersection of the two constraints whose slopes bracket the objective function's slope.

MARGINAL PROFIT

Marginal Profit = Marginal Revenue – Marginal Cost

For a linear objective function, marginal profit is constant — it equals the coefficient of that decision variable.

Marginal Analysis — Definitions Recap

Marginal Cost (MC): the additional cost of producing one more unit; the rate of change of cost with output.

Marginal Revenue (MR): the additional revenue from selling one more unit; in a linear objective function, MR is simply the coefficient of that decision variable, and stays constant.

6 CALCULATION ROADMAP

General path for any LP question

1- Read Question

Identify the decision variables and limits

2- Build Constraints

Translate wording into inequalities

3- Plot & Shade

Find intercepts, shade the feasible region

4- Find Corners

Solve intersecting lines simultaneously

5- Optimize & Check

Test corners in objective function, state answer

7 STEP-BY-STEP METHODS

Kept separate — do not mix methods

METHOD A · Formulating an LP Problem

- 1 Define a variable for each product/outcome to be optimized
- 2 Write one constraint per limiting resource, using $\leq$ for maximums, $\geq$ for minimums
- 3 Add non-negativity constraints $x, y \geq 0$
- 4 Write the objective function to maximize or minimize

METHOD C · Corner Point Theorem

- 1 Identify every corner point of the feasible region
- 2 Solve pairs of intersecting constraint lines simultaneously for each corner's (x,y)
- 3 Substitute each corner's (x,y) into the objective function
- 4 Select the corner giving the best (max/min) result

METHOD B · Graphical Method (Plotting)

- 1 For each constraint, set $x=0$ to find the y-intercept, and $y=0$ to find the x-intercept
- 2 Plot and join the two intercepts for each line
- 3 Test the origin in each inequality to decide which side to shade
- 4 The overlapping shaded area is the feasible region

METHOD D · Slope Comparison Method

- 1 Rearrange the objective function and each constraint into $y = \dots + (\text{slope})x$
- 2 Rank the slopes in order
- 3 Find the two constraints whose slopes bracket the objective function's slope
- 4 Solve those two constraints simultaneously for the optimum (x,y)

METHOD E · Testing for Redundancy / Unboundedness / No Feasible Region

- 1 Plot all constraints and identify the feasible region
- 2 Remove one constraint at a time — if the region is unchanged, it is redundant
- 3 If the feasible region is open-ended in the direction of optimization, the LP is unbounded
- 4 If no region satisfies every constraint together, there is no feasible solution

8 DIFFERENT CASES / SCENARIOS

Recognise the wording, pick the right route

IF wording says "cannot exceed" / "no more than" / maximum
→ USE $\leq$ in the constraint

IF wording says "at least" / "no less than" / minimum
→ USE $\geq$ in the constraint

IF a range is given (e.g. "5 to 7 units")
→ Write **two inequalities** ($\geq$ lower, $\leq$ upper)

IF the objective is to maximize (e.g. profit)
→ Optimum is the corner **furthest** from the origin

IF the objective is to minimize (e.g. cost)
→ Optimum is the corner **closest** to the origin

IF removing a constraint leaves the feasible region unchanged
→ That constraint is **redundant**

IF the objective function can improve without limit in the feasible region
→ The LP is **unbounded** — no finite solution

IF no combination satisfies every constraint at once
→ There is **no feasible region / solution**

9 COMPARISON TABLES

Don't confuse these!

	Corner Point Theorem	Slope Comparison Method
Approach	Test every corner's (x,y) directly in the objective function	Compare the objective function's slope with each constraint's slope
When useful	Few corner points; straightforward substitution	Quickly narrows down which two constraints bound the optimum
Result	Directly gives the best objective function value	Identifies which two lines to solve simultaneously for the optimum

	Redundancy	Unboundedness
Definition	A constraint that does not affect the feasible region	No limit to improving the objective value within the feasible region
Effect on feasible region	No change	Infinite extension in the direction of optimization
Implication	Can be removed; simplifies the model	No finite optimal solution exists — model needs revising

Constraint wording	Sign to use	Example
Maximum limit	$\leq$	Maximum sales demand of 5,000 units $\rightarrow x \leq 5,000$
Minimum limit	$\geq$	Supply at least 2,000 units $\rightarrow x \geq 2,000$
Non-negativity	≥ 0	Cannot make a negative number of products $\rightarrow x, y \geq 0$

10 GRAPH INTERPRETATION

Reading a linear programming graph

Reading the feasible region

The shaded (often double- or triple-shaded) area is the feasible region. Any (x,y) inside or on its boundary satisfies every constraint. The straight-line boundary of each constraint is its outer limit (maximum constraints) or inner limit (minimum constraints).

Reading corner points

Corner points are where boundary lines of the feasible region cross. The optimum solution always lies at one of these corners — never in the middle of the shaded area.

Reading the objective function line

Since the objective function's value (C) does not affect its slope, it can be plotted at any convenient value. Moving this line outward (maximizing) or inward (minimizing), parallel to itself, shows which corner it last touches — that corner is the optimum.

No feasible region

If, after plotting and shading, the constraints leave no overlapping shaded area, the problem has no feasible solution — there is no combination of x and y that satisfies every constraint at once.

11 WORKED CALCULATION LOGIC

Method students must reproduce in the exam

GIVEN → FORMULA → SUBSTITUTION → CALCULATION → ANSWER

A company makes Product X (contribution \$8) and Product Y (contribution \$12). Direct labour: $3x+2y \leq 20,000$; Machine time: $x+2.5y \leq 12,000$; sales demand $x \leq 6,000$, $y \leq 4,000$. Maximize $C=8x+12y$.

GIVEN: Objective $C=8x+12y$; constraints as above; corner points found by solving intersecting lines simultaneously.

FORMULA: Corner point theorem — substitute each corner's (x,y) into $C=8x+12y$

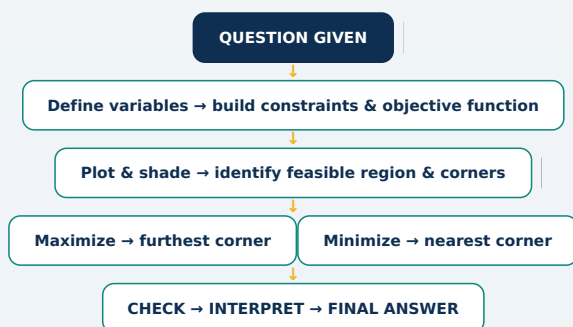
SUBSTITUTION / CALCULATION: Solve each pair of intersecting constraints for their corner (x,y), then compute C at each corner

FINAL ANSWER: Compare all corner values of C — the corner giving the highest C is the optimal product mix

12 EXAM QUESTION PATTERNS

- **Formulation** — write constraints & objective function from a word problem
- **Graph reading** — match a shaded region to its set of inequalities (or vice versa)
- **Corner point calculation** — solve simultaneous equations for a corner
- **Optimization** — maximize profit/contribution or minimize cost
- **Redundancy / unboundedness / no feasible solution** identification
- Marginal cost/revenue/profit calculation questions

14 QUICK DECISION TREE



13 EXAM TRAPS

⚠ WATCH OUT

✗ **$\leq$ vs $\geq$ confusion** — "no more than" is $\leq$, "at least" is $\geq$; reversing them flips the feasible region.

✗ **Forgetting non-negativity** ($x,y \geq 0$) — without it, the feasible region can wrongly extend into other quadrants.

✗ **Shading the wrong side** — always test the origin (or another point, if the line passes through the origin).

✗ **Maximize/minimize mix-up** — picking the nearest corner for a maximization problem, or vice versa.

✗ **Treating a redundant constraint as binding** — check whether removing it actually changes the feasible region.

CHAPTER 4 IN 60 SECONDS

6 steps: variables→constraints→plot→feasible region→objective→optimum

Max limit: $\leq$ **Min limit:** $\geq$

Shade test: plug in origin (0,0)

Corner point theorem: test each corner in Z

Slope method: objective slope sits between two constraint slopes

Max → furthest corner; **Min** → nearest corner

Redundant: removable, no effect on region

Marginal Profit = MR – MC