

1 CHAPTER OVERVIEW

Your visual roadmap through the chapter

Sequences

Ordered lists of terms; arithmetic vs geometric sequences

1

Geometric Progression

Common ratio, nth term, sum of n terms

2

Infinite Geometric Series

Sum to infinity when the common ratio is less than 1

3

WHAT IT'S ABOUT

Numerical sequences formed by a definite law — arithmetic progression (constant difference) and geometric progression (constant ratio) — and how to find any term or the sum of a series.

WHY IT MATTERS FOR QAB

Progressions model recurring business situations — salary increments, loan installments, savings plans, and growth at a constant rate — and underpin later financial mathematics topics.

EXAM SKILLS REQUIRED

Identify AP vs GP; apply the nth term and sum formulas; solve for an unknown term count (n) including via the quadratic formula; evaluate sum to infinity.

2 KEY DEFINITIONS

Source-based terminology, exam-ready

Sequence

A succession of terms arranged in a definite order and formed according to a definite law, i.e. the set $\{u_1, u_2, u_3, \dots, u_n, \dots\}$.

Key word: definite order

Series

The expression $u_1 + u_2 + u_3 + \dots + u_n$, i.e. the sum of all terms of a sequence; symbolically expressed as $\sum u_n$.

Key word: sum of terms

Term

Each successive number forming a sequence; denoted $T_1, T_2, \dots, T_n$ where T_n is the nth or "general" term.

Key word: T_n = general term

Arithmetic Progression (A.P.)

A series in which the successive terms increase or decrease by a constant common difference (d).

Key word: constant difference

Common Difference (d)

The constant amount obtained by subtracting any term of an A.P. from the term immediately following it.

Key word: d = subtract

Geometric Progression (G.P.)

A sequence where the ratio between a term and the one immediately preceding it is constant throughout; neither the first term nor the ratio can be zero.

Key word: constant ratio

Common Ratio (r)

The constant ratio linking consecutive terms of a G.P.; found by dividing a term by the term preceding it.

Key word: r = divide

Infinite Series

A sequence with no last term. As n approaches infinity, r^n approaches zero if $|r| < 1$, allowing a finite sum to be found.

Key word: no last term

3 CORE CONCEPT

Explained in exam-oriented language

CONCEPT → Arithmetic Progression

MEANING

Each term is linked to the previous term by adding/subtracting a constant (d).

PURPOSE

Model quantities that change by a fixed amount each period.

APPLICATION

Salary increments, fixed-step savings plans, loan installments rising by a flat amount.

CONCEPT → Geometric Progression

MEANING

Each term is linked to the previous term by multiplying by a constant (r).

PURPOSE

Model quantities that grow or shrink by a fixed percentage each period.

APPLICATION

Interest earned, depreciation over time, installments rising by a fixed %, infinite-series totals.

4 KEY RULES & PRINCIPLES

Every source rule, exam-ready

RULE 1 nth term relationship (A.P.)

- The coefficient of d is always one less than the term number
- 2nd term = $a+d$, 3rd term = $a+2d$, ... nth term = $a+(n-1)d$

RULE 2 Choosing the sum formula (A.P.)

- Use $S = n/2 (a+l)$ when the first term AND last term are known
- Use $S = n/2 [2a+(n-1)d]$ when the first term AND common difference are known
- With any 3 of the 5 quantities (a, d, l, n, S) known, the other 2 can be found

RULE 3 Representing unknown A.P./G.P. terms

- A.P. 3 terms: $(a-d), a, (a+d)$ | 4 terms: $(a-3d), (a-d), (a+d), (a+3d)$
- G.P. 3 terms: $a/r, a, ar$ | 4 terms: $a/r^3, a/r, ar, ar^3$

RULE 4 Properties of an A.P. under transformation

- Add/subtract a constant k to every term → still an A.P., same common difference d
- Multiply every term by k → still an A.P., new common difference = $d \times k$
- Divide every term by k → still an A.P., new common difference = $d \div k$

RULE 5 Choosing the sum formula (G.P.)

- If $r < 1$, use $S = a(1-r^n) \div (1-r)$ — more convenient
- If $r > 1$, use $S = a(r^n-1) \div (r-1)$ — more convenient
- If $r = 1$, every term equals a, so $S = n \times a$

RULE 6 Sum to infinity — existence condition

- Exists ONLY if $|r| < 1$ (r between -1 and 1)
- As $n \rightarrow \infty$, $r^n \rightarrow 0$, so the r^n term can be ignored
- If $r \geq 1$, the sum grows without limit — no finite sum to infinity

5 FORMULA ZONE

Every formula supported by the source text

A.P. — NTH TERM

$$T_n = a + (n-1)d$$

a = first term, **d** = common difference, **n** = term number. **T_n** is also called "**l**" (the last term) when it's the final term of a finite series.

A.P. — SUM OF N TERMS

$$S = n/2 (a + l) \quad \text{or} \quad S = n/2 [2a + (n-1)d]$$

First form needs first term (a) & last term (l); second form needs first term (a) & common difference (d).

ARITHMETIC MEAN (A.M.)

$$A.M. = (a + b) \div 2$$

The single value A that sits between two quantities a and b so that a, A, b form an A.P.

G.P. — NTH TERM

$$T_n = a r^{n-1}$$

a = first term, **r** = common ratio, **n** = term number. Neither a nor r can be zero.

G.P. — SUM OF N TERMS

$$S = a(1-r^n) \div (1-r) \quad [r < 1] \quad | \quad S = a(r^n-1) \div (r-1) \quad [r > 1]$$

Both formulas are mathematically equivalent; pick whichever avoids a negative denominator for easier calculation.

SUM TO INFINITY OF A G.P.

$$S_\infty = a \div (1-r)$$

Only valid when $|r| < 1$. **a** = first term, **r** = common ratio.

Representing Unknown Terms (for solving simultaneous problems)

A.P. (3 terms): $(a-d), a, (a+d)$ **A.P. (5 terms):** $(a-2d), (a-d), a, (a+d), (a+2d)$

G.P. (3 terms): $a/r, a, ar$ **G.P. (5 terms):** $a/r^2, a/r, a, ar, ar^2$

6 CALCULATION ROADMAP

General path for any progression question

1- Read Question

Identify given terms/values

2- Identify Type

Constant difference (A.P.) or constant ratio (G.P.)?

3- Select Formula

nth term, sum, or sum to infinity

4- Substitute

Put known values into the formula

5- Calculate & Check

Solve, discard invalid roots, state answer

7 STEP-BY-STEP METHODS

Kept separate — do not mix methods

METHOD A · Find the nth Term of an A.P.

- 1 Identify the first term (a) and common difference (d)
- 2 Identify which term number (n) is required
- 3 Substitute into $T_n = a + (n-1)d$
- 4 Calculate and state the final term value

METHOD D · Find the nth Term & Sum of a G.P.

- 1 Identify the first term (a) and find r by dividing a term by the one before it
- 2 Substitute into $T_n = ar^{n-1}$ for the required term
- 3 Choose the sum formula matching whether $r < 1$ or $r > 1$
- 4 Substitute and calculate the sum S

METHOD B · Find the Sum of an A.P.

- 1 Identify a, and either d or the last term (l)
- 2 Choose the matching sum formula (Rule 2)
- 3 Substitute the known values
- 4 Calculate the sum S

METHOD E · Sum to Infinity of a G.P.

- 1 Find the common ratio r
- 2 Check $|r| < 1$ — if not, sum to infinity does not exist
- 3 Substitute a and r into $S_\infty = a \div (1-r)$
- 4 Calculate the final sum

METHOD C · Solve for n (Unknown Number of Terms)

- 1 Substitute known a, d and S into $S = n/2[2a + (n-1)d]$
- 2 Multiply out and rearrange into standard quadratic form
- 3 Solve using the quadratic formula
- 4 Discard the negative/invalid root; keep the sensible n

Reading a Sequence (visual)

A.P. example 2, 6, 10, 14 → each term is **+4** from the last (add a constant).
G.P. example 16, 8, 4, 2 → each term is **$\times \frac{1}{2}$** of the last (multiply by a constant).

8 DIFFERENT CASES / SCENARIOS

Recognise the pattern, pick the right route

IF each term differs by a fixed amount added/subtracted

→ Sequence is an **A.P.** — use nth term / sum formulas

IF each term differs by a fixed multiplying factor

→ Sequence is a **G.P.** — use ratio-based formulas

IF first term (a) and last term (l) are both known

→ USE $S = \frac{n}{2} (a+l)$

IF first term (a) and common difference (d) are known

→ USE $S = \frac{n}{2} [2a + (n-1)d]$

IF solving for n produces a quadratic equation

→ Solve normally, then **discard the negative/nonsensical root**

IF the G.P. common ratio $|r| < 1$

→ **Sum to infinity exists:** $S_{\infty} = a \div (1-r)$

IF the G.P. common ratio $|r| \geq 1$

→ **Sum to infinity does not exist** (grows without limit)

IF asked to find 3/4/5 unknown terms of a progression

→ Represent symmetrically around a: e.g. A.P. $(a-d), a, (a+d)$

9 COMPARISON TABLES

Don't confuse these!

	Arithmetic Progression (A.P.)	Geometric Progression (G.P.)
Link between terms	Add/subtract a constant (d)	Multiply by a constant (r)
nth term	$T_n = a + (n-1)d$	$T_n = ar^{n-1}$
Sum of n terms	$S = \frac{n}{2}(a+l)$ or $\frac{n}{2}[2a + (n-1)d]$	$S = \frac{a(1-r^n)}{(1-r)}$ or $\frac{a(r^n-1)}{(r-1)}$
Sum to infinity	Not applicable	Exists only if $ r < 1$: $S_{\infty} = a/(1-r)$

G.P. Sum — Case	Condition	Result
Case I	$r = 1$	Every term equals a; $S = n \times a$
Case II	$r < 1$	r^n shrinks toward 0 as n grows; $S = \frac{a(1-r^n)}{(1-r)}$, and $S_{\infty} = a/(1-r)$ exists
Case III	$r > 1$	r^n grows without bound; $S = \frac{a(r^n-1)}{(r-1)}$, and S_{∞} is infinite (does not exist)

Properties of an A.P. under transformation (recap)

Transformation applied to every term	New common difference
Add or subtract a constant k	Same as original (d)
Multiply by a constant k	$d \times k$
Divide by a constant k	$d \div k$

10 WORKED CALCULATION LOGIC

Method students must reproduce in the exam

GIVEN → FORMULA → SUBSTITUTION → CALCULATION → ANSWER

A company funds an 18-month expansion: Rs. 100,000 in month 1, increasing Rs. 10,000 per month. Find the 18th payment and the total invested.

GIVEN: $a = 100,000$, $d = 10,000$, $n = 18$ (A.P.)

FORMULA: $T_n = a + (n-1)d$ then $S = n/2[2a + (n-1)d]$

SUBSTITUTION: $T_{18} = 100,000 + (17)(10,000)$ | $S = 18/2[2(100,000) + (17)(10,000)]$

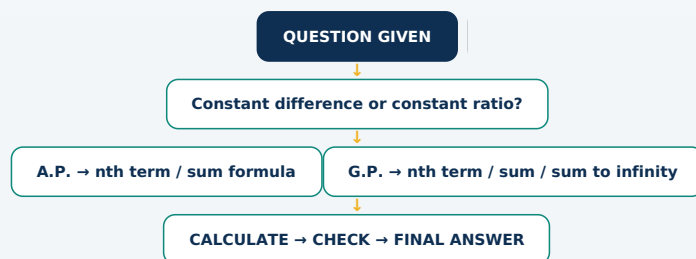
CALCULATION: $T_{18} = 100,000 + 170,000 = 270,000$ | $S = 9[200,000 + 170,000] = 9(370,000)$

FINAL ANSWER: 18th payment = Rs. 270,000; total invested = Rs. 3,330,000

12 EXAM QUESTION PATTERNS

- **Identification** — is a given sequence an A.P. or a G.P.?
- **nth term** — direct formula application
- **Sum calculation** — choosing the correct sum formula
- **Solve-for-n** — rearranging into a quadratic equation
- **Sum to infinity** — existence check then calculation
- Business application word problems — salary, loans, savings, depreciation

14 QUICK DECISION TREE



13 EXAM TRAPS

⚠ WATCH OUT

✗ **Misidentifying A.P. vs G.P.** — check whether terms ADD or MULTIPLY by a constant.

✗ **Wrong sum formula** — using $n/2(a+l)$ when l is unknown, instead of the d -based formula.

✗ **Keeping a negative n** — the number of terms must always be positive; discard the invalid root.

✗ **Applying sum to infinity when $|r| \geq 1$** — it simply does not exist in that case.

✗ **Common ratio sign errors** — dividing terms in the wrong order or mishandling a negative ratio.

CHAPTER 3 IN 60 SECONDS

A.P. nth term: $a + (n-1)d$

A.P. sum: $n/2(a+l)$ or $n/2[2a + (n-1)d]$

G.P. nth term: ar^{n-1}

G.P. sum: $a(1-r^n)/(1-r)$ [$r \neq 1$]

Sum to infinity: $a/(1-r)$, only if $|r| < 1$

A.M.: $(a+b)/2$

Solve for n → quadratic, discard negative root

Trap: A.P. adds, G.P. multiplies