

## 1 CHAPTER OVERVIEW

Your visual roadmap through the chapter

### Binomial

Fixed trials, 2 outcomes, constant probability

1

### Hyper-geometric

Sampling without replacement

2

### Poisson

Events at a constant average rate

3

### Normal

Continuous, bell-shaped;  $\pm$  approximations

4

#### WHAT IT'S ABOUT

Theoretical probability distributions used to estimate the probability of a given result when a sample is taken from a population with known characteristics.

#### WHY IT MATTERS FOR QAB

Choosing and applying the right distribution is central to quality control, risk estimation and forecasting problems in business.

#### EXAM SKILLS REQUIRED

Recognise which distribution applies; calculate probabilities using each formula; use Poisson/Normal tables; apply the normal approximation with continuity correction.

## 2 KEY DEFINITIONS

Source-based terminology, exam-ready

### Probability Distribution

Similar to a frequency distribution but using probabilities instead of frequencies; the total always equals 1.

**Key word:** totals to 1

### Probability Function

A formula giving the probability of a random variable taking a specific value.

**Key word:** formula for  $P(x)$

### Theoretical vs Experimental

Theoretical distributions give expected probability when outcomes are equally likely; experimental distributions result from actual observed occurrences.

**Key word:** expected vs observed

### Discrete vs Continuous Distribution

Discrete distributions record a finite number of outcomes; continuous distributions record an infinite number of possibilities (e.g. time, distance).

**Key word:** finite vs infinite

### Binomial Distribution

Applies to a discrete variable with a fixed number of independent trials, each with two outcomes and a known, constant probability of success.

**Key word:** fixed trials, constant  $p$

### Hyper-geometric Distribution

Applies when a sample is randomly selected WITHOUT replacement from a population with two types of items; trials are dependent.

**Key word:** no replacement

### Poisson Distribution

A discrete distribution for events that occur randomly, independently, singly, and at a constant average rate, over a period of time or space.

**Key word:** rate, not probability

### Normal Distribution & Z-score

A symmetric, bell-shaped continuous distribution. The Z-score (standardized normal variate) measures how many standard deviations a value is from the mean.

**Key word:**  $Z = (x - \mu) / \sigma$

### 3 CORE CONCEPT

Explained in exam-oriented language

#### CONCEPT → Discrete Distributions

##### MEANING

Binomial, Hyper-geometric and Poisson each model a countable number of "successes" under different conditions.

##### APPLICATION

Quality control (defect counts), sampling problems, event-rate forecasting.

##### PURPOSE

Estimate the probability of a specific count of outcomes.

#### CONCEPT → Normal Distribution

##### MEANING

A continuous, symmetric bell-shaped distribution fitting many naturally occurring variables.

##### APPLICATION

Heights, weights, product dimensions; also approximates Binomial/Poisson when conditions are met.

##### PURPOSE

Estimate the probability of a value falling in a given range using Z-scores and tables.

### 4 KEY RULES & PRINCIPLES

Conditions for each distribution to apply

#### RULE 1 Binomial — 4 conditions

- Fixed number of trials; each trial independent
- Each trial has exactly two outcomes (success/failure)
- Probability of success is known and CONSTANT for every trial (implies replacement, or a population so large the effect is negligible)

#### RULE 3 Poisson — 4 conditions

- Events occur randomly, independently, and singly (one at a time)
- Events happen, on average, at a constant rate ( $\lambda$ )
- Models HOW OFTEN something happens, not the probability of one trial

#### RULE 2 Hyper-geometric — 4 conditions

- Outcomes classified success/failure; sample taken WITHOUT replacement
- Probability of success CHANGES each trial — trials are dependent
- The experiment is repeated a definite number of times

#### RULE 4 Normal distribution & approximations

- Normal curve: symmetric, bell-shaped; mean = median = mode; total area = 1
- Normal approximates Binomial when  $0.1 < p < 0.9$  and  $np > 5$
- Normal approximates Poisson when  $\lambda > 30$
- Poisson can approximate Binomial when  $n$  is large and  $p$  is small
- A continuity correction ( $\pm 0.5$ ) is required whenever a continuous distribution approximates a discrete one

## 5 FORMULA ZONE — DISCRETE DISTRIBUTIONS

Every formula supported by the source text

### BINOMIAL DISTRIBUTION

$$P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$p$ =P(success),  $q$ =P(failure)= $1-p$ ,  $n$ =sample size. Mean= $np$ ; SD= $\sqrt{npq}$ .

### HYPER-GEOMETRIC DISTRIBUTION

$$P(x) = \frac{[kCx \times (N-k)C(n-x)]}{NCn}$$

$N$ =population size,  $n$ =sample size,  $k$ =possible successes in population,  $x$ =successes in sample.

### POISSON DISTRIBUTION

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$\lambda$ =average rate;  $e$ =2.71828. Mean= $\lambda$ = $np$ ; SD= $\sqrt{\lambda}$ .

### POISSON RECURSIVE SHORTCUT

$$P(n) = P(n-1) \times \lambda \div n$$

Each successive probability = previous probability  $\times \lambda/n$ .  $P(x=0)$  is always  $e^{-\lambda}$ .

### Choosing Between "or fewer" and "at least" (Poisson & Binomial ranges)

"**x or fewer**": sum the individual probabilities of 0 up to  $x$  (or read directly from a cumulative table).

"**at least x**" (= **x or more**): since all probabilities sum to 1, calculate  $1 - P(\text{fewer than } x)$  instead of summing an open-ended list.

## 5 FORMULA ZONE — NORMAL DISTRIBUTION

Every formula supported by the source text

### Z-SCORE (STANDARDIZED NORMAL VARIATE)

$$Z = (x - \mu) \div \sigma$$

$x$ =value under investigation,  $\mu$ =mean,  $\sigma$ =standard deviation. Converts any normal distribution to the standard one (mean 0, SD 1).

### NORMAL APPROXIMATION TO BINOMIAL

$$\text{Mean} = np \quad | \quad \text{SD} = \sqrt{np(1-p)}$$

Valid when  $0.1 < p < 0.9$  and  $np > 5$ . Requires continuity correction.

### NORMAL APPROXIMATION TO POISSON

$$\text{Mean} = \lambda \quad | \quad \text{SD} = \sqrt{\lambda}$$

Valid when  $\lambda > 30$ . Requires continuity correction.

### KEY AREAS UNDER THE NORMAL CURVE

$$\mu \pm 1\sigma = 68\% \quad | \quad \mu \pm 2\sigma = 95.5\% \quad | \quad \mu \pm 3\sigma = 99.7\%$$

Also:  $\pm 1.96\sigma$  encloses 95%;  $\pm 2.58\sigma$  encloses 99% of the total distribution.

### Continuity Correction Table (discrete event → corrected normal range)

| Discrete event                  | Normal-approximation correction |
|---------------------------------|---------------------------------|
| Exactly 10 successes: $P(x=10)$ | $P(9.5 \leq x \leq 10.5)$       |
| More than 10: $P(x > 10)$       | $P(x > 10.5)$                   |
| 10 or more: $P(x \geq 10)$      | $P(x > 9.5)$                    |
| Fewer than 10: $P(x < 10)$      | $P(x < 9.5)$                    |
| 10 or fewer: $P(x \leq 10)$     | $P(x < 10.5)$                   |

## 6 CALCULATION ROADMAP

General path for any distribution question

### 1- Read Question

Note trials, replacement, and what's asked

### 2- Identify Distribution

Binomial, Hyper-geometric, Poisson, or Normal?

### 3- Select Formula

Direct calc, table lookup, or approximation

### 4- Substitute

Put known values into the formula

### 5- Calculate & Check

Solve, recheck, state final answer

## 7 STEP-BY-STEP METHODS

Kept separate — do not mix methods

### METHOD A · Binomial Probability

- 1 Confirm the 4 binomial conditions are met; identify  $n$ ,  $p$ ,  $q$ ,  $x$
- 2 Calculate the binomial coefficient  $n! \div [x!(n-x)!]$
- 3 Multiply by  $p^x$  and  $q^{n-x}$  ( $p$  to the  $x$ ,  $q$  to the  $n-x$ )
- 4 For a range, sum individual  $P(x)$  values or use the complement

### METHOD D · Normal Distribution (Find a Probability)

- 1 Identify the mean ( $\mu$ ), standard deviation ( $\sigma$ ), and value(s)  $x$
- 2 Calculate  $Z = (x - \mu) \div \sigma$  for each value
- 3 Look up the  $Z$  value in the normal table (area for half the curve)
- 4 Add/subtract 0.5 or combine areas depending on what's asked (above/below/between)

### METHOD B · Hyper-geometric Probability

- 1 Identify  $N$  (population),  $n$  (sample),  $k$  (successes in population),  $x$  (successes wanted)
- 2 Calculate  $kCx$  and  $(N-k)C(n-x)$  separately
- 3 Multiply those two together
- 4 Divide by  $NCn$

### METHOD E · Normal Distribution (Find a Value from %)

- 1 Identify the target proportion/percentile
- 2 Look up the matching  $Z$  value in the normal table (reverse lookup)
- 3 Rearrange  $Z = (x - \mu) / \sigma$  to  $x = \mu \pm Z\sigma$
- 4 Substitute  $\mu$ ,  $\sigma$  and  $Z$  to find  $x$

### METHOD C · Poisson Probability

- 1 Identify  $\lambda$  (average rate) for the period/space asked about — rescale if needed
- 2 Calculate  $e^{-\lambda}$  once
- 3 Substitute into  $P(x) = e^{-\lambda} \lambda^x / x!$ , or use the recursive shortcut for consecutive  $x$  values
- 4 For "at least", use  $1 - P(\text{fewer})$

### METHOD F · Normal Approximation (Binomial or Poisson)

- 1 Check the approximation condition is met ( $0.1 < p < 0.9$  &  $np > 5$ , or  $\lambda > 30$ )
- 2 Calculate the approximating mean and SD
- 3 Apply the continuity correction ( $\pm 0.5$ ) to match the wording
- 4 Calculate  $Z$  and read off the probability as in Method D

## 8 DIFFERENT CASES / SCENARIOS

Recognise the situation, pick the right distribution

**IF trials are fixed, independent, WITH replacement (or a huge population)**

→ USE the **Binomial** distribution

**IF sampling is WITHOUT replacement from a small/finite population**

→ USE the **Hyper-geometric** distribution

**IF you're counting random events over time/space at a constant rate**

→ USE the **Poisson** distribution

**IF the variable is continuous and roughly bell-shaped/symmetric**

→ USE the **Normal** distribution

**IF n is large and p is small (Binomial hard to compute directly)**

→ Approximate with **Poisson**

**IF n is large and  $0.1 < p < 0.9$  with  $np > 5$**

→ Approximate the Binomial with the **Normal** (+ continuity correction)

**IF  $\lambda$  (Poisson rate) is greater than 30**

→ Approximate the Poisson with the **Normal** (+ continuity correction)

**IF asked for "at least"/"more than" x**

→ USE the complement:  $1 - P(\text{fewer than/up to } x)$

## 9 COMPARISON TABLES

Don't confuse these!

|                               | Binomial                    | Hyper-geometric    | Poisson                             |
|-------------------------------|-----------------------------|--------------------|-------------------------------------|
| <b>Replacement</b>            | With (or negligible effect) | Without            | N/A — models a rate                 |
| <b>Trials</b>                 | Independent, fixed n        | Dependent, fixed n | Independent, over time/space        |
| <b>Probability of success</b> | Constant (p)                | Changes each trial | Governed by constant rate $\lambda$ |

|                          | Discrete Distributions (Binomial/Hyper-geo/Poisson) | Normal Distribution                                                  |
|--------------------------|-----------------------------------------------------|----------------------------------------------------------------------|
| <b>Variable type</b>     | Discrete (countable)                                | Continuous                                                           |
| <b>Output</b>            | Exact $P(x)$ for a whole-number count               | Area/probability over a range, via Z-score                           |
| <b>Link between them</b> | —                                                   | Normal can approximate Binomial/Poisson (with continuity correction) |

| Normal Approximation | Condition                    | Mean & SD Used                         |
|----------------------|------------------------------|----------------------------------------|
| <b>To Binomial</b>   | $0.1 < p < 0.9$ and $np > 5$ | Mean= $np$ , SD= $\sqrt{np(1-p)}$      |
| <b>To Poisson</b>    | $\lambda > 30$               | Mean= $\lambda$ , SD= $\sqrt{\lambda}$ |

## 10 READING TABLES & THE NORMAL CURVE

How to interpret the source's tools

### Reading a Poisson table

Columns are values of  $\lambda$ ; rows are values of  $x$ . The cell gives  $P(x)$  directly for that combination — no calculation needed. A cumulative Poisson table instead gives  $P(x \text{ or fewer})$ .

### Reading the Normal (Z) table

The table gives the area between the mean and Z standard deviations — i.e. HALF the distribution only. Combine areas (add or subtract 0.5) depending on whether the question asks above/below/between values.

### Reading the normal curve shape

A higher standard deviation gives a flatter, more spread-out curve; a lower standard deviation gives a taller, narrower curve. All normal curves are symmetric about the mean regardless of shape.

### Combining Z areas for a range

For a value between two points straddling the mean, ADD the two individual Z areas. For two points on the same side of the mean, SUBTRACT the smaller Z area from the larger.

## 11 WORKED CALCULATION LOGIC

Method students must reproduce in the exam

### BINOMIAL: GIVEN → FORMULA → SUBSTITUTION → ANSWER

10% of homes have an occupant aged 90+. What is the probability that a sample of 10 homes contains exactly 2 such homes?

**GIVEN:**  $n=10$ ,  $p=0.1$ ,  $q=0.9$ ,  $x=2$

**FORMULA:**  $P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$

**SUBSTITUTION:**  $P(2) = \frac{10!}{2!(8!)} \times 0.1^2 \times 0.9^8 = 45 \times 0.01 \times 0.4305$

**ANSWER:**  $P(2) = 0.1937$  or **19.37%**

### NORMAL: GIVEN → FORMULA → SUBSTITUTION → ANSWER

Bags of sugar: mean=1,010g, SD=5g. What's the probability a bag weighs less than 1,000g?

**GIVEN:**  $x=1,000$ ,  $\mu=1,010$ ,  $\sigma=5$

**FORMULA:**  $Z = (x - \mu) / \sigma$

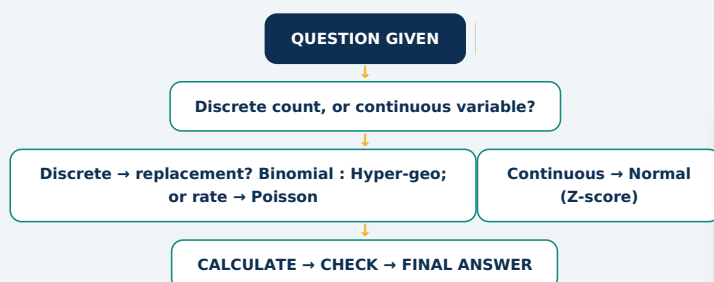
**SUBSTITUTION:**  $Z = (1,000 - 1,010) / 5 = -2$ . Table: area for  $Z=2$  is 0.4772

**ANSWER:**  $P(x < 1,000) = 0.5 - 0.4772 = 0.0228$  or **2.28%**

## 12 EXAM QUESTION PATTERNS

- **Identification** — which distribution applies to a described scenario
- **Direct calculation** — Binomial, Hyper-geometric or Poisson  $P(x)$
- **Table reading** — Poisson or Normal Z tables
- **Normal probability** — using Z-scores for a range
- **Reverse lookup** — finding  $x$  from a given percentage
- Normal approximation with continuity correction

## 14 QUICK DECISION TREE



## 13 EXAM TRAPS

### ⚠ WATCH OUT

✗ **Wrong distribution choice** — check replacement (Binomial) vs no replacement (Hyper-geometric) carefully.

✗ **Forgetting the continuity correction** when using Normal to approximate a discrete distribution.

✗ **Reading the Z table as the full probability** — it only gives HALF the distribution's area.

✗ **Sign/direction errors** — a negative Z means below the mean; check which side the question needs.

✗ **"At least" vs "more than"** — these use different correction boundaries (9.5 vs 10.5); check the continuity correction table.

### CHAPTER 12 IN 60 SECONDS

**Binomial:** replacement, constant  $p$ , mean= $np$

**Hyper-geo:** no replacement, dependent trials

**Poisson:** constant rate  $\lambda$ , mean= $\lambda$

**Normal:**  $Z = (x - \mu) / \sigma$ , table=half curve

**Approx Binomial→Poisson:**  $n$  large,  $p$  small

**Approx →Normal:**  $np > 5$  or  $\lambda > 30$

**Continuity correction:**  $\pm 0.5$

**"At least  $x$ "** =  $1 - P(\text{fewer than } x)$