

1 CHAPTER OVERVIEW

Your visual roadmap through the chapter

Possible Outcomes

Tree diagrams, counting principle, permutations & combinations

1

Probability

Classical probability, addition & multiplication laws, conditional probability

2

WHAT IT'S ABOUT

Determining the number of possible outcomes for a situation (using tree diagrams, permutations and combinations), and measuring the likelihood that an event occurs (probability).

WHY IT MATTERS FOR QAB

Estimating outcomes and likelihoods underpins prediction, risk assessment and decision-making in business situations.

EXAM SKILLS REQUIRED

Apply the counting principle, permutations and combinations; calculate probability using classical and relative-frequency approaches; apply addition, multiplication and conditional-probability rules; use probability tables and trees.

2 KEY DEFINITIONS

Source-based terminology, exam-ready

Permutation

A set of several possible ways in which a number of things can be ordered or arranged — an "ordered combination" where order matters.

Key word: order matters

Combination

A group of items chosen from a larger number without regard for their order — order does NOT matter.

Key word: order doesn't matter

Factorial (n!)

An instruction to multiply all whole numbers counting down from n to 1 (e.g. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$). Remember: $0! = 1$.

Key word: $0! = 1$

Probability

A measure of the likelihood of something happening; a number between 0 (cannot occur) and 1 (certain to occur).

Key word: 0 to 1

Outcome / Event / Trial

An outcome is what happens as a result of an event. An event is a group of one or more outcomes. A trial is a single attempt to obtain a given event.

Key word: outcome $\subseteq$ event

Mutually Exclusive Events

Events where the occurrence of one prevents the occurrence of the other in a single observation — no common outcomes.

Key word: no overlap

Independent Events

Events where the outcome of one does not affect the outcome of the other.

Key word: no effect on each other

Conditional Probability

The probability of an event occurring given that another event has already occurred (one event's probability depends on another).

Key word: given that...

3 CORE CONCEPT

Explained in exam-oriented language

CONCEPT → Possible Outcomes

MEANING

Counting how many ways a set of choices can be arranged or selected.

PURPOSE

Find the total number of arrangements or selections without listing every one.

APPLICATION

Product variations, team/committee selection, code/password combinations.

CONCEPT → Probability

MEANING

A number from 0 to 1 measuring how likely an event is to occur.

PURPOSE

Support predictions and decisions under uncertainty.

APPLICATION

Risk assessment and risk management in business.

4 KEY RULES & PRINCIPLES

Every source rule, exam-ready

RULE 1 Counting principle (multiplicative)

- If there are m ways of doing one thing and n ways of doing another (independent), there are $m \times n$ ways of doing both
- Extends to three or more events: $m \times n \times p$

RULE 3 Permutations with repeated items

- Divide $n!$ by $x!$ for each group of identical/repeated items
- With two repeated groups, divide by both $x_i!$ and $x_{ij}!$

RULE 2 Additive principle

- If m and n are the number of ways two mutually exclusive events can occur, the possibility that either occurs is $m + n$

RULE 4 Permutations vs Combinations

- Use permutations (nPr) when the order of selection matters
- Use combinations (nCr) when order does NOT matter

5 FORMULA ZONE — COUNTING, PERMUTATIONS & COMBINATIONS

Every formula supported by the source text

PERMUTATIONS (ALL n ITEMS)

$$n!$$

n = number of items; arrange all of them, order matters.

PERMUTATIONS WITH IDENTICAL ITEMS

$$n! \div x!$$

x = number of identical items/repetitions.

PERMUTATIONS OF SIZE r (nPr)

$$P_{n,r} = n! \div (n-r)!$$

Choosing and arranging r items out of n , without repetition, order matters.

COMBINATIONS (nCr)

$$C_{n,x} = n! \div [x!(n-x)!]$$

Also called the binomial coefficient. x =size of the group selected; order does not matter.

5 FORMULA ZONE — PROBABILITY LAWS

Every formula supported by the source text

CLASSICAL PROBABILITY

$$P(\text{event}) = \text{favourable outcomes} \div \text{total possible outcomes}$$

Used when all outcomes are equally likely.

RELATIVE FREQUENCY

$$\text{Relative Frequency} = \text{trials where event occurred} \div \text{total trials}$$

Used when outcomes are NOT all equally likely — based on observation/experiment.

ADDITION LAW (OVERLAPPING EVENTS)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Use when A and B can both occur together (not mutually exclusive).

ADDITION LAW (MUTUALLY EXCLUSIVE)

$$P(A \text{ or } B) = P(A) + P(B)$$

Use when A and B cannot both occur (no common outcomes).

MULTIPLICATION LAW (INDEPENDENT)

$$P(A \text{ and } B) = P(A) \times P(B)$$

Use when one event's outcome does not affect the other.

MULTIPLICATION LAW (DEPENDENT) & CONDITIONAL PROBABILITY

$$P(A \text{ and } B) = P(A) \times P(B|A) \quad | \quad P(B|A) = P(A \text{ and } B) \div P(A)$$

$P(B|A)$ = probability of B occurring given A has occurred.

COMPLEMENT RULE

$$P(\text{not } A) = 1 - P(A)$$

The probability of an event plus the probability of it not occurring always equals 1. Useful for "at least one" questions.

6 CALCULATION ROADMAP

General path for any probability question

1- Read Question

Identify events & what's asked



2- Identify Relationship

Exclusive/overlapping?
Independent/dependent?



3- Select Formula

Addition, multiplication, or
conditional law



4- Substitute

Put known probabilities into the
formula



5- Calculate & Check

Solve, recheck, state final answer

7 STEP-BY-STEP METHODS

Kept separate — do not mix methods

METHOD A · Counting Principle / Tree Diagram

- 1 List each independent choice and how many options it has
- 2 Confirm the choices are independent of each other
- 3 Multiply all the option counts together
- 4 For mutually exclusive alternative routes, add instead

METHOD D · Basic Probability

- 1 Check whether outcomes are equally likely
- 2 If yes: $P(\text{event}) = \text{favourable} \div \text{total possible outcomes}$
- 3 If not (based on observation): use relative frequency instead
- 4 Express as a decimal or a percentage

METHOD B · Permutations

- 1 Count total items (n) and, if relevant, the size chosen (r)
- 2 Arranging all n items: use $n!$
- 3 Some items identical: divide $n!$ by $x!$ (per repeated group)
- 4 Choosing/arranging r of n : use $n! \div (n-r)!$

METHOD E · Addition Law (OR)

- 1 Decide if events A and B can occur together (overlapping) or not (mutually exclusive)
- 2 Mutually exclusive: $P(A \text{ or } B) = P(A) + P(B)$
- 3 Overlapping: subtract the shared outcomes, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- 4 State the final probability

METHOD C · Combinations

- 1 Confirm order does NOT matter (a group/selection, not an arrangement)
- 2 Identify n (total items) and x (size of the group selected)
- 3 Substitute into $nCx = n! \div [x!(n-x)!]$
- 4 For multi-category selections, multiply the separate nCx values together

METHOD F · Multiplication Law & Conditional Probability (AND)

- 1 Decide if A and B are independent or dependent
- 2 Independent (e.g. with replacement): $P(A \text{ and } B) = P(A) \times P(B)$
- 3 Dependent (e.g. without replacement): $P(A \text{ and } B) = P(A) \times P(B|A)$
- 4 For "at least one", use the complement: $1 - P(\text{none})$

METHOD G · Multi-Stage Problems (Tables & Trees)

- 1 Set up a probability table, contingency table, or probability tree for the stages/groups given
- 2 Multiply along each branch/path for a joint probability
- 3 Add across relevant branches/cells for a combined probability
- 4 For a conditional question, divide the relevant joint probability by its row/column total

8 DIFFERENT CASES / SCENARIOS

Recognise the situation, pick the right route

IF choices are independent stages of one task

→ USE the **counting principle**: multiply the option counts

IF there are alternative, mutually exclusive routes

→ USE the **additive principle**: add the option counts

IF the order of selection matters

→ USE a **permutation** ($n!$ or nPr)

IF the order of selection does NOT matter

→ USE a **combination** (nCx)

IF some items being arranged are identical

→ Divide $n!$ by $x!$ for each repeated group

IF events A and B can both happen (overlap)

→ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

IF one event's outcome does NOT affect the other

→ USE **$P(A \text{ and } B) = P(A) \times P(B)$** (independent)

IF a selection is made WITHOUT replacement

→ Events are **dependent** — use $P(A) \times P(B|A)$

9 COMPARISON TABLES

Don't confuse these!

	Permutation	Combination
Order	Matters	Does not matter
Formula	$n!$ or $n! \div (n-r)!$	$n! \div [x!(n-x)!]$
Example	Arranging books on a shelf in a given sequence	Choosing a committee/team from a group

	Mutually Exclusive Events	Independent Events
Meaning	Cannot both occur; no common outcomes	The outcome of one does not affect the other
OR rule	$P(A \text{ or } B) = P(A) + P(B)$	N/A (this concerns AND, not OR)
AND rule	Cannot both occur, so $P(A \text{ and } B) = 0$	$P(A \text{ and } B) = P(A) \times P(B)$

Multi-stage Tool	What it is	Best used for
Probability table	Sets out probabilities of different combinations directly	A small number of categories/stages
Contingency table	Like a probability table, scaled up by a convenient total (e.g. 300 items)	Visualising outcomes as whole numbers of items
Probability tree	A diagram of branching links between possible probabilities	Multi-stage problems with sequential events

10 READING TABLES & TREES

How to interpret the source's tools

Reading a probability tree

Each branch shows a probability; multiply along a path (branch × branch) for the joint probability of that path. Add the results of every path leading to the outcome you want. All final path probabilities must sum to 1.

Reading a contingency/two-way table

Rows and columns represent two variables (e.g. department × age group). A cell is the count/probability for that combination. Row and column totals give the marginal counts used for simple probabilities.

Conditional probability from a table

To find $P(B|A)$, take the cell for "A and B" and divide it by A's row (or column) total — not the grand total.

Reading combined probability + counting problems

When a question needs both a count (nCx) and a probability, first find the total number of equally likely selections (nCx as the denominator), then count the favourable selections (as the numerator).

11 WORKED CALCULATION LOGIC

Method students must reproduce in the exam

GIVEN → FORMULA → SUBSTITUTION → CALCULATION → ANSWER

There are 4 white and 5 blue pens. What is the probability that two white pens are drawn respectively, if neither pen is replaced?

GIVEN: 4 white, 5 blue (9 total); draws are without replacement (dependent events)

FORMULA: $P(A \text{ and } B) = P(A) \times P(B|A)$

SUBSTITUTION: $P(\text{1st white}) = 4/9$; $P(\text{2nd white} | \text{1st white}) = 3/8$

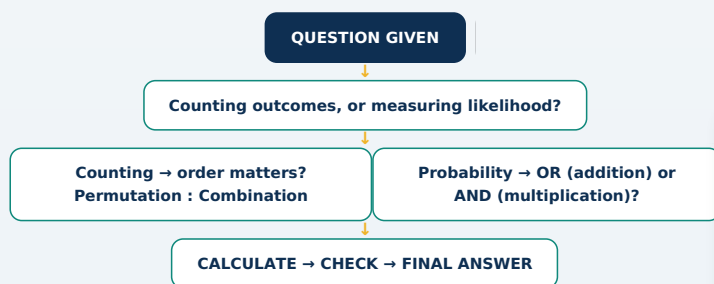
CALCULATION: $P(A \text{ and } B) = 4/9 \times 3/8 = 12/72$

FINAL ANSWER: $P(\text{both white}) = 1/6 \approx 0.167$ or **16.7%**

12 EXAM QUESTION PATTERNS

- **Counting** — total arrangements/combinations for a scenario
- **Permutation/combinatation calculation** — direct formula application
- **Basic probability** — classical or relative-frequency calculation
- **Addition/multiplication law** — OR / AND probability questions
- **Conditional probability** — with/without replacement scenarios
- Multi-stage problems using tables or probability trees

14 QUICK DECISION TREE



13 EXAM TRAPS

⚠ WATCH OUT

- ✗ **Permutation vs combination mix-up** — check whether order genuinely matters before choosing the formula.
- ✗ **Forgetting $-P(A \text{ and } B)$** when events overlap — only skip it for genuinely mutually exclusive events.
- ✗ **Using $P(A) \times P(B)$ for dependent events** instead of $P(A) \times P(B|A)$ — check for "without replacement".
- ✗ **Forgetting $0! = 1$** in factorial-based calculations.
- ✗ **"At least one" questions** — direct calculation is error-prone; use the complement $1 - P(\text{none})$ instead.

CHAPTER 11 IN 60 SECONDS

Counting: $m \times n$ (independent), $m+n$ (exclusive)
nPr: $n!/(n-r)!$ **nCx:** $n!/[x!(n-x)!]$
P(event) = favourable/total
OR (overlap): $P(A) + P(B) - P(A \text{ and } B)$
OR (exclusive): $P(A) + P(B)$
AND (independent): $P(A) \times P(B)$
AND (dependent): $P(A) \times P(B|A)$
Trap: "at least one" = $1 - P(\text{none})$