

1 CHAPTER OVERVIEW

Your visual roadmap through the chapter

Expressions & Equations 1

Terms, variables, constants, coefficients, degree of a polynomial

Solving Linear Equations 2

One-variable & simultaneous two-variable equations; word problems

Quadratic Equations 3

Standard form, graph shape, roots as x-intercepts

Solving Quadratics 4

Factorisation, completing the square, the quadratic formula

WHAT IT'S ABOUT

Building & solving algebraic equations of degree 1 (linear) and degree 2 (quadratic), including systems of two linear equations, using factorisation, completing the square and the quadratic formula.

WHY IT MATTERS FOR QAB

Equation-solving underpins later chapters — coordinate geometry, linear programming, financial mathematics and break-even/optimisation problems all build directly on this chapter.

EXAM SKILLS REQUIRED

Translate word problems into equations; solve simultaneous equations by elimination; factorise, complete the square, and apply the quadratic formula accurately.

2 KEY DEFINITIONS

Source-based terminology, exam-ready

Expression

A number, a variable, or a combination of numbers and variables with operation symbol(s); separated into terms by operations.

Key word: terms

Equation

Formed when two mathematical expressions are considered equal, shown by the sign "=".

Key word: equality

Variable / Constant

Variable: a value that may change within the scope of the problem (x, y, z). Constant: a value that remains unchanged, even if unknown (a, b, c).

Key word: changes vs fixed

Coefficient

A multiple attached to a variable within an equation (e.g. the 2 in $y = 2x + c$).

Key word: multiplier

Degree (of a term/equation)

In a polynomial, the sum of all exponents of the variables used; the degree of an equation is its highest power.

Key word: highest power

Linear Equation

Each term is a constant or the product of a constant and a single variable; no term is raised to a power or root. Degree = 1.

Key word: degree one

Dependent vs Independent Variable

y is the dependent variable (changes because of x); x is the independent variable. By convention, the dependent variable sits alone on the left of "=".

Key word: y depends on x

Quadratic Expression

An expression in which the degree of the variable is two (the highest power is a square).

Key word: degree two

3 CORE CONCEPT

Explained in exam-oriented language

CONCEPT → Linear Equations

MEANING

Degree-one equation; standard straight-line form $y = a + mx$. Model relationships that change at a constant rate.

APPLICATION

Single unknown problems and simultaneous two-variable systems solved by elimination.

PURPOSE

CONCEPT → Quadratic Equations

MEANING

Degree-two equation, standard form $y = ax^2 + bx + c$. Find the two roots (x-values) where the curve crosses $y = 0$.

APPLICATION

Area/distance, forecasting, interest, price-product modelling, optimisation (min/max).

PURPOSE

4 KEY RULES & PRINCIPLES

Every source rule, exam-ready

RULE 1 Solving a linear equation

- Read the question; note requirements & given information
- List the unknown variable(s), assign a letter (x)
- Express the relationship between variables as an equation
- Solve the equation for the unknown
- Recheck the solution

RULE 3 Simultaneous linear equations

- Manipulate one equation so a variable's coefficient matches the other equation
- Add or subtract the equations to eliminate that variable
- Solve for the remaining variable
- Substitute back into an original equation to find the other variable

RULE 2 Formulating from word problems

- Step 1: Pick the smaller/unknown value as x
- Step 2: Write other information in terms of x
- Step 3: Cancel out written values; keep remaining info
- Step 4: Formulate the equation and solve
- Step 5: Check the solution

RULE 4 Discriminant ($b^2 - 4ac$) conditions

- If $b^2 - 4ac > 0$ → roots are real and unequal
- If $b^2 - 4ac < 0$ → roots are imaginary / complex
- If $b^2 - 4ac = 0$ → roots are real and equal
- Not every quadratic can be factorised — use completing the square or the formula instead

5 FORMULA ZONE

Every formula supported by the source text

STRAIGHT LINE (LINEAR EQUATION)

$$y = a + mx$$

y = dependent variable x = independent variable a = y-intercept m = slope/gradient

QUADRATIC EQUATION — STANDARD FORM

$$y = ax^2 + bx + c$$

a, b, c are constants; $a \neq 0$. Plots as a U-shape ($a > 0$) or inverted-U ($a < 0$).

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use when $ax^2 + bx + c = 0$. Constant must be on the RHS → general form first. Automatically "completes the square" for you.

PERFECT-SQUARE IDENTITIES (FOR COMPLETING THE SQUARE)

$$(a+b)^2 = a^2 + 2ab + b^2 \quad (a-b)^2 = a^2 - 2ab + b^2$$

Reference identities used to rewrite $ax^2 + bx$ as a perfect square, then solve for x.

6 CALCULATION ROADMAP

General path for any equation question

1- Read Question

Identify given information

2- Identify Unknown

Assign a letter (x)

3- Select Method

Formula, factorisation, elimination

4- Substitute

Put values into equation/formula

5- Calculate & Check

Solve, recheck, state final answer

7 STEP-BY-STEP METHODS

Kept separate — do not mix methods

METHOD A · Simultaneous Linear Equations (Elimination)

- 1 Multiply one/both equations so a variable's coefficient matches
- 2 Add or subtract the equations to eliminate that variable
- 3 Solve the single remaining equation for the other variable
- 4 Substitute back into an original equation to find the first variable

METHOD B · Factorisation (coefficient of $x^2 = 1$)

- 1 Write in standard form $x^2 + bx + c = 0$
- 2 Find two numbers that sum to b and multiply to c
- 3 Write as $(x \pm ?)(x \pm ?) = 0$
- 4 Set each bracket = 0 and solve for x

METHOD C · Factorisation (coefficient of $x^2 > 1$)

- 1 Multiply coefficient of x^2 by the constant ($a \times c$)
- 2 Find two numbers summing to b, multiplying to ($a \times c$)
- 3 Rewrite the x-term as two new x-terms using those numbers
- 4 Bracket first two & last two terms (flip sign inside bracket if needed)
- 5 Factor each bracket separately to find a common factor
- 6 Complete the factorisation; set each bracket = 0 and solve

METHOD D · Completing the Square

- 1 Rearrange $ax^2 + bx + c = 0$ into $ax^2 + bx = -c$
- 2 Divide throughout by a (coefficient of x^2 must be 1)
- 3 Take half the coefficient of x (with its sign)
- 4 Square that number
- 5 Add it to both sides of the equation
- 6 Rearrange the LHS into a perfect square and solve for x

METHOD E · The Quadratic Formula

- 1 Write the equation in general form $ax^2 + bx + c = 0$
- 2 Identify a, b, and c (with correct signs)
- 3 Substitute into $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- 4 Evaluate the discriminant first, then both root values

8 DIFFERENT CASES / SCENARIOS

Recognise the situation, pick the right route

IF the equation has one variable, degree 1

→ USE **the 5-step linear solving approach** (isolate x)

IF two linear equations share two unknowns

→ USE **elimination** (match coefficients, add/subtract)

IF the quadratic factorises easily (small integers)

→ USE **factorisation** — fastest method

IF the quadratic does not factorise neatly

→ USE **completing the square** or the **quadratic formula**

IF discriminant $b^2 - 4ac > 0$

→ Roots are **real and unequal** (curve cuts x-axis twice)

IF discriminant $b^2 - 4ac = 0$

→ Roots are **real and equal** (curve touches x-axis once)

IF discriminant $b^2 - 4ac < 0$

→ Roots are **imaginary/complex** (curve never touches x-axis)

IF a word problem gives two unknowns, one described relative to the other

→ Let the **smaller value** = x, express the other in terms of x

9 COMPARISON TABLES

Don't confuse these!

	Linear Equation	Quadratic Equation
Degree	One (highest power = 1)	Two (highest power = 2)
Standard form	$y = a + mx$	$y = ax^2 + bx + c$
Graph shape	Straight line	U-shape or inverted-U (parabola)
Number of roots	One root (single-variable case)	Up to two roots (x-axis crossings)

	Factorisation	Completing the Square	Quadratic Formula
When to use	When factors are easy to spot	When it cannot be factorised neatly	Always works; used when other methods are difficult
Core idea	Reverse of expanding brackets	Rewrite as a perfect square using $(a \pm b)^2$ identities	Formula that auto-completes the square
Limitation	Not every quadratic factorises	Only works cleanly once coefficient of $x^2 = 1$	None — but requires correct a, b, c signs

10 GRAPH INTERPRETATION

Reading the quadratic curve

Shape rule

If $a > 0$ → graph is **U-shaped**. If $a < 0$ → graph is an **inverted U**. Example: $x^2 - 8x + 12 = y$ cuts the x-axis at $x = 2$ and $x = 6$ — these are the equation's two roots.

Reading the roots

Roots = the x-values where the curve crosses the line $y = 0$ (the x-axis). Not every quadratic crosses the x-axis — if it doesn't, the equation is insoluble in real numbers.

Method students must reproduce in the exam

FINAL ANSWER: $x \approx -6.19$ or $x \approx 387.81 \rightarrow$ break-even at approximately 388 units

13 EXAM TRAPS

- ### WATCH OUT

x Mismatched coefficients in elimination — multiply the whole equation, not just one term.

CALCULATE → CHECK → FINAL ANSWER

Trap: always flip sign inside brackets